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(Beta & Gamma Function)

Beta Function, B

$$B(m,n) = \int_0^1 (x)^{m-1} (1-x)^{n-1} dx.$$

$$B(m,n) = B(n,m)$$

$$B(m,n) = \int_0^\infty \frac{(x)^{m-1}}{(1+x)^{m+n}} dx$$

$$B(m,n) = \frac{\lfloor m-1 \ \lfloor n-1}{\lfloor m+n-1}$$

$$B(6,7) = \int_0^1 (x)^5 (1-x)^6 dx$$

$$\int_0^1 (x)^{3/2} (1-x)^7 dx \Rightarrow B\left(\frac{5}{2}, 8\right)$$

$$\int_0^\infty \frac{(x)^4}{(1+x)^7} \Rightarrow \int_0^\infty \frac{(x)^{5-1}}{(1+x)^{5+2}}$$

$$B(5,2)$$

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$$\int_0^1 (x)^{1/2} (1-x)^{3/2}\, dx$$

$$\Rightarrow \beta\left(\frac{3}{2}, \frac{5}{2}\right)$$

Gamma function

<u>Gamma</u> $\Gamma(n)$

$$\Gamma(n) = \int_0^\infty (x)^{n-1} e^{-x}\, dx$$

$$\Gamma(n) = (n-1)\,\Gamma(n-1)$$

$$\Gamma(n) = \underline{|n-1} \Rightarrow (n-1)!$$

$$\beta(m,n) = \frac{\Gamma(m)\;\Gamma(n)}{\Gamma(m+n)}$$

$$\Gamma\left(\frac{1}{2}\right) = \underline{\sqrt{\pi}}$$

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$$\int_0^\infty e^{-ax} (x)^n \, dx.$$ → Gamma function

$ax = z$

$x = \frac{z}{a}$

$dx = \left(\frac{dz}{a}\right)$

$$\int_0^\infty e^{-z} \left(\frac{z}{a}\right)^n \left(\frac{dz}{a}\right).$$

$$\int_0^\infty e^{-z} \frac{(z)^n \, dz}{a^{n+1}}.$$

$$\frac{1}{a^{n+1}} \int_0^\infty e^{-z} (z)^n \, dz.$$

$$\left(\frac{1}{a^{n+1}}\right) \Gamma(n+1)$$ Ans

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$$\int_0^\infty e^{-ax} (x)^{n-1} dx = \frac{\Gamma(n)}{a^n}$$

$$ax = z$$

$$x = \left(\frac{z}{a}\right)$$

$$dx = \frac{dz}{a}$$

$$\int_0^\infty e^{-z} \left(\frac{z}{a}\right)^{n-1} \frac{dz}{a}$$

$$\frac{1}{a^n} \int_0^\infty e^{-z} (z)^{n-1} dz$$

$$\left(\frac{1}{a^n}\right) \Gamma(n) \Rightarrow \left(\frac{\Gamma(n)}{a^n}\right) \text{Ans}$$

$$\int_0^1 y^{q-1}\left(\lg\frac{1}{y}\right)^{p-1} dy \Rightarrow \frac{\Gamma(p)}{q^p}$$

$$\lg\frac{1}{y} = z$$

$$\frac{1}{y} = \frac{e^{z}}{1} \Rightarrow 1$$

$$y = e^{-z}$$

$$dy = -e^{-z}\,dz$$

$$\int_\infty^0 (e^{-z})^{q-1} (z)^{p-1} (-e^{-z})\,dz$$

$$(e^{-z})^{q-1+1}$$

$$\therefore \int_\infty^0 e^{-zq} (z)^{p-1}\,dz$$

$$\int_0^\infty e^{-zq} (z)^{p-1}\,dz$$

$$zq = t$$

$$z = \frac{t}{q}$$

$$dz = \frac{dt}{q}$$

$$\int_0^\infty e^{-t} \frac{(t)^{p-1}}{(q)^{p-1}} \times \frac{dt}{q}$$

$$\frac{1}{q^p}\int_0^\infty e^{-t} (t)^{p-1}\,dt \Rightarrow \left(\frac{1}{q^p}\right) \Gamma(p)$$

$$\int_0^1 \frac{dx}{\sqrt{1-x^4}} \cancel{dx} = \frac{\sqrt{\pi}}{4} \frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}$$

$$\int_0^1 \frac{dx}{(1-x^4)^{1/2}}$$

$x^4 = z$

$x = (z)^{1/4}$

$dx = \frac{1}{4}(z)^{\frac{1}{4}-1} dz$

$$\int_0^1 \frac{1}{4}(z)^{\frac{1}{4}-1} (1-z)^{-1/2} dz$$

$$\frac{1}{4} \beta\left(\frac{1}{4}, \frac{1}{2}\right)$$

m, n

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$\frac{1}{4} \frac{\Gamma(\frac{1}{4})\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{4})}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\frac{1}{4} \sqrt{\pi} \frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}$$ Hence Proved

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$$\int_0^\infty (x)^{p-1} e^{-kx}\, dx$$

$kx = z$

$x = \left(\frac{z}{k}\right)$

$dx = \frac{dz}{k}$

$$\int_0^\infty \left(\frac{z}{k}\right)^{p-1} e^{-z} \frac{dz}{k}$$

$$\frac{1}{k^p} \int_0^\infty e^{-z} (z)^{p-1}\, dz$$

$$\left(\frac{1}{k^p}\right) \Gamma(p) \quad \text{Hence Prov}$$

$$\int_0^\infty \frac{(x)^{m-1}}{(a+bx)^{m+n}}\,dx = \frac{B(m,n)}{a^n b^m}$$

$$\int_0^\infty \frac{(x)^{m-1}\,dx}{(a+bx)^{m+n}}$$

$bx = az$

$b\,dx = a\,dz$

$dx = \frac{a}{b}\,dz$

$$\int_0^\infty \frac{\left(\frac{az}{b}\right)^{m-1} \frac{a\,dz}{b}}{(a+az)^{m+n}}$$

$$\Rightarrow \int_0^\infty \frac{\frac{(a)^m}{b^m}\,(z)^{m-1}\,dz}{(a)^{m+n}\,(1+z)^{m+n}}$$

$$\frac{a^m}{b^m\,(a)^{m+n}} \int_0^\infty \frac{(z)^{m-1}\,dz}{(1+z)^{m+n}}$$

$$\frac{1}{b^m a^n}\,B(m,n)$$ Hence Prove

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(1) $$\int_0^\infty e^{-k^2x^2}\,dx$$

(2) $$\int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy$$

(3) $$\int_0^\infty (x)^{2n-1} e^{-ax^2}\,dx$$

(4) $$\int_0^1 x^m (\log x)^n\,dx$$

(5) $$\int_0^\infty (x)^3 e^{-x^3}\,dx$$